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UNITED STATES DEPARTMENT OF AGRICULTURE
BUREAU OF AGRICULTURAL ENGINEERING
Drainage Division

ANNOTATED BIBLIOGRAPHY ON THE FLOW OF WATER AROUND BENDS

(Arranged chronologically)

By

David L. Yarnell, Senior Drainage Engineer

Washington, D.C. July, 1936

C. Bossut. Nouvelles Experiences sur la Resistance des Fluides. 1777.

———— Traite Theoretique et Experimental d'Hydrodynamique. Paris, 1786.

P.L.G. Dubuat. Principes Hydrauliques, Paris, 1786. Part 1, Chapter VII, pp.141-151. Part 1 - 453 p. illus. Part 2 - 402 p. illus.

Made about 25 experiments on a pipe 1-inch in diameter and 117 inches long. Concluded that the "head due to the resistance in any bent tube depends on the number of deflections between the entrance and the departure." Derived the formula

$$\text{Head lost} = \frac{V^2 s^2 n}{m}$$

in which s = sine of the angle of rebound, v = speed of the current, m = a constant number = 3000 in French inches, n = number of rebounds.

J.B. Venturi. Experimental Inquiries Concerning the Principle of the Lateral Communication of Motion in Fluids, Applied to the Explanation of Various Hydraulic Phenomena. 1797. Abstracted from the Translation by William Nicholson, London, April 5, 1799, as given in Thomas Tredgold's "Tracts on Hydraulics, London, 1836." 219 p., illus.

Experiment 23.- The two tubes ABC, DEF, Fig. 14, Plate II, are 15 inches long; their diameter is 14.5 lines (1.3 inches). The conical portions A, D, have the form of the contraction of the vein of fluid, and are applied to the orifice P, Fig. 1, which is 18 lines in diameter, with 32.5 inches depth, or charge of superincumbent fluid. The elbows, or flexures, BC, EF, are made in the plane of the horizon. These two pipes are made of copper soldered with silver, and the workmanship carefully executed. The curvature BC was drawn out or bended into the form of a quarter of a circle, by filling the tube with melted lead in order that it might preserve its diameter during the act of bending. The elbow DEF is constructed in a right angle. The expenditure through these two tubes was compared with that afforded through a right-lined cylindrical tube of similar dimensions and in like circumstances. The four cubical feet of water flowed out of the cylindrical tube in 45 seconds; out of the curved tube ABC in 50 seconds; and out of the angular tube DEF in 70 seconds.

T. Young. Hydraulic investigations subservient to an intended Croonean lecture on the motion of the blood. Royal Soc. of London, Philosophical Transactions for 1808, v. 98, pp. 164-186.

Mentions tests of Couplet, Bossutt, DuBuat and Gerstner. First conceived the necessity of taking into consideration the length of the curve and the radius of curvature. In the 25 experiments made by DuBuat he rejected 10 in framing his formula, and the remaining 15 agreed with it very closely.

Young found that all but five experiments agreed with his (Young's) theory. No statement could be found telling for what h_b really stands.

Dr. Young found

$$h_b = \frac{0.000006\,\phi\,\rho^{1/8}V^2}{\rho} \quad \text{(in English feet units)}$$

where ϕ is the number of degrees in the curve, ρ = the radius of the curve, h_b = the head due to the resistance of the curve, and v = the velocity in feet per second.

John Robinson. A System of Mechanical Philosophy, Edinburgh, 1822, Vol. 11, Sec. 4, pages 488-496.

Discusses at considerable length DuBuat's experiments and gives a practical application of the formula.

George Rennie. On the friction and resistance of fluids. Philosophical Transactions of the Royal Soc., of London, v.121, 1831, pp. 323-442.

Made series of tests on a lead pipe 1/2-inch in diameter and 15 feet long. Curves used were $3\frac{1}{4}$ and $7\frac{1}{2}$ inches radius. Gives results of his tests, calls attention to the incorrectness of both DuBuat's and Young's formulas but does not deduce any of his own.

J.F. D'Aubuisson De Voisins. A Treatise on Hydraulics, Translated by Joseph Bennett, D. Van Nostrand, New York, 1857, 532p. illus.

Gives Bossut's experiments on bent pipe, also Rennie's tests, and DuBuat's experiments. Gives DuBuat formula and an example applying the same.

A. A. Humphreys and H.L. Abbott. Report upon the Physics and Hydraulics of the Mississippi River. Professional Paper No. 4, of the Corps of Topographical Engineers, United States Army, 1861, 456 p., illus.

Gives DuBuat's formula $h = \dfrac{V^2 \sin^2 \alpha}{2998.5}$

in French inches which reduced to English feet becomes

$$h = \frac{V^2 \sin^2 \alpha}{266.3}$$

From their experiments made in 1851 and 1858 at Vicksburg and on other bends between Baton Rouge and Carrollton they found that DuBuat's English unit formula gave too small values and that the formula should be

$$H = \frac{V^2 \sin^2 \alpha}{134}$$

M. De Saint Venant. Memoire sur l'influence retardatrice de la courbure dans les courants d'eau, Comptes Rendus, v.54, pp. 38-42. (1862)

Quotes DuBuat's formula and gives data on several tests with calculated and observed losses.

Julius Weisbach. Die Experimental Hydraulik, 1863; Der Civil Enginieur, v. IX, 1863; Der Civil Enginieur, v. X, 1864; Theoretical Mechanics, Translated by E.B. Coxe, 1899, p.897.

From his own experiments and from the results of some observations made by DuBuat, the author has deduced the following empirical formula for the coefficients of resistance of water in passing through bent pipes.

For bends with circular cross-sections,

$$\zeta = 0.131 + 1.847 \left(\frac{a}{r}\right)^{7/2}$$

He then gives for various ratios of $\frac{a}{r}$, values of the coefficient ζ

a = coefficient of contraction.
r = radius of curvature.
ζ = coefficient of resistance.

W.J.M. Rankine. Applied Mechanics, Chas. Griffin & Co., London, Ed. 5, 1870, 648 p., illus.

Quotes Weisbach's formula but contains practically no discussion on the subject.

Lewis D.A. Jackson. Hydraulic Manual and Statistics. W.H. Allen Co., London, 1875, Ed. 3, pp. 55-57.

Quotes a formula, apparently DuBuat's, and gives Humphrey's and Abbott's modification of DuBuat's formula. Also quotes Weisbach's formula for bends of pipes.

John Neville. Hydraulic Tables, Coefficients and Formulae. Ed. 3, Lockwood & Co., London, 1875. 494 p., illus.

"..... resistance due to bends are independent of the pressure."

"The experiments of Bossut, DuBuat, and others, show that the loss of head from bends and curves - like that from friction - increases as the square of the velocity; but when the curves have large radii, and the bends are very obtuse, the loss is very small. With a head of nearly 3 feet, Venturi's twenty-third experiment, when reduced, gives - for a short straight tube 15 inches long, and $1\frac{1}{4}$ inches in diameter, having the junction of the form of the contracted vein - very nearly 0.873 for the coefficient of discharge."

"Mr. Mallett's experiments with a siphon tube 6 inches by $1\frac{1}{2}$ inches, about 3 feet long, suited for weir crests and a straight tube of the same dimensions every way gives coefficients from 0.860 to 0.874 due to the bend."

"Neither DuBuat nor Young took any notice of the relation that must exist between the resistance and the ratio of the radius of curvature to the radius of the pipe. Weisbach does, and combining DuBuat's experiments with some of his own, 'derived a formula'".

"Weisbach does not find the loss of head in a right angular bend greater than 0.984 $\frac{V^2}{2g}$; while Venturi's twenty-third experiment, made with extreme care, shows the loss to be 1.876 $\frac{V^2}{2g}$."

"It appears therefore that the amount of the velocity of the water moving directly toward the bend must be taken into consideration."

M. Boussinesq. Essai sur la theorie des - eaut courantes. Memoires A.L'Academie des Sciences 23, no. 1, 68p.

James Thomson. Proceedings of the Royal Society, May 4, 1876; Proceedings of the Royal Society, No. 182, 1877, p.356; Proceedings Institution of Mechanical Engineers, Aug. 6, 1879; Collected Papers in Physics and Engineering of Prof. James Thomson, by Sir Joseph Larmor and James Thomson, 1912.

"On the Origin of Windings of Rivers in Alluvial Plains, with Remarks on the Flow of Water Round Bends in Pipes." Pages 96-100. (From the Proceedings of the Royal Society, May 4, 1876).

James Thomson.(Cont'd) "People overlook the hydraulic principle, not generally known, that a stream flowing along a straight channel and thence into a curve must flow with a diminished velocity along the outer bank, and an increased velocity along the inner bank." Then "Why does not the inner bank wear away more than the other one?" Professor Thomson advances his theory on the spiral flow or oblique motion toward the inner bank of the bottom filaments. "It is especially worthy of notice that the oblique flow along the bottom towards the inner bank begins even upstream from the bend."

"From Biographical Sketch in 'Collected Papers', page LXXIV. Professor Thomson made on his brother's lecture room table a clay model of a river bend. Turning on the water, he found the particles at the bottom of the channel were carried across to the inner bank as he said they would go. A wooden model was made for the exhibition in Kelingrove Museum which was prepared for the British Association. He had intended to clear away the original clay model after showing it to his brother. But when Sir William saw the experiment, he insisted on keeping the clay model as it was, in order to let members of the Association see the experiment and hear the explanation from the author himself. A short communication describing the experiments was sent afterwards to the Royal Society. Later, on the suggestion of Prof. Barr, the course of the stream lines was indicated by little specks of aniline dye introduced so as to adhere to the bed of the channel at various places."

"Experimental Demonstration in respect to the Origin of Windings of Rivers in Alluvial Plains, and to the Mode of Flow of Water Round Bends of Pipes."

(From the Proceedings of the Royal Society, No. 182, 1877, p. 356.)

In this paper Prof. Thomson mentions the use of threads attached to pins standing upright like thin posts in the river, the threads showing the direction of the current. He also used sand, peas, clover and poppy seed to show the direction of the current. No sketches are shown. The channel used was 8 inches wide and between 1 and 2 inches deep.

"On the Flow of Water Round River Bends" (From the Proceedings of the Institution of Mechanical Engineers, read at a meeting in Glasgow, Aug. 6, 1879).

In this article the sketches usually shown in text books are presented, and also the same description as presented above.

Osborne Reynolds. On Certain Laws Relating to Rivers and Estuaries. Papers on Mechanical and Physical Subjects, 1887, v.2, p. 329.

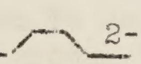
"But, as was first pointed out by Dr. James Thomson as affording an explanation of the generally observed fact that the beds of rivers are scoured on their convex sides and silted on their concave, the layers of water adjacent to the bed do not always move in the general direction of the stream. There are often steady cross currents at the bottom, as in the case mentioned, though such cross currents do not exist except under circumstances which may be readily distinguished. The most important of these is that pointed out by Dr. Thomson - curvature in the general direction of the stream, in which case the centrifugal force of the more rapidly moving water above overbalances that of the water retarded by the bottom and forces the latter back towards the center of the curve."

"This action is universal, where even the lateral boundaries are such as to require the water to move in curved streams; the drift at the bottom does not follow the general direction of the stream, but sets towards the center of the curve."

John R. Freeman. Experiments Relating to the Hydraulics of Fire Streams. Trans. Amer. Soc. Civ. Engin. v. 21, 1889.

Gives some experiments on resistance caused by bends in fire hose.

Tests made on bends like the sketches.

45°  2-ft. rad.  2-ft. rad.  2,3,& 4-ft. rad.

"..... within these limits there was more loss with the large radius than with the small.

J.T. Fanning. Practical Treatise on Hydraulic and Water Supply Engineering. D. Van Nostrand, New York, Ed. 11. 1893, 644 p.

Quotes Weisbach's formula and also gives coefficients for resistance to flow.

Mentions experiments of DuBuat and Venturi.

M. Leliavski. Currents in Streams and the Formation of Stream Beds. 6th Internat'l Congress of Internal Navigation, The Hague, The Netherlands, 1894.

Discusses flow around bends, describes secondary currents in open channel bends and shows drawings of apparatus used in measuring the secondary currents.

Thomas Box. Practical Hydraulics. Spon, Ltd., London, 1899 80 p., illus.

Gives Weisbach's formula.

Gardner S. Williams. The Foundation of our Knowledge of Hydraulic Curve Resistance. Michigan Technic, 1899, p.48.

Quotes, DuBuat's tests as reported by D'Aubuisson gives data of Rennie's experiments, also Weisbach's experiments as published in "Die Experimental Hydraulic" and Der Civilingenieur, Vols. IX and X, and Freeman's Tests on Fire Hose.

M.E. Sullivan. New Hydraulics. Mining Reporter Press, Denver, 1900, 301.p.

Sullivan advances a new formula which is

$$h = \frac{A\, r\, V^2}{90\, R\, 2g} = \frac{AV^2 \; .007764}{90R}$$

in which r = 1/2 diameter of pipe.
R = radius of central arc of bend in diameters of pipe.
A = number of degrees of the arc of bend.
V = mean velocity of flow.

Quotes Weisbach's formula and Rankine's changed form of the Weisbach formula. Also mentions Rennie's tests on a leaden pipe.

Shows table of comparison of head lost from Weisbach's and his own formula and then states that they may both be low.

N.D. Tiapkin. Apparatus for Measuring Velocities and Discharges in Open Channels (Rivers and Canals). Published by the Author, Professor in the Moscow Imperial Engineering Institute, Bachmetiewskaya, 15, Moscow, Russia, 8 Vol., 30 plates, 1901.

N.D. Tiapkin. (Cont'd)

Experiments of Leliavski described in this work are given in Engineering News, Vol. 52, 1904, pp.183-186.

C.W.L. Alexander.

The Resistance Offered to the Flow of Water in Pipes by Bends and Elbows. Proc. Inst. Civ. Engin., v. 159, 1902, pp. 341-364.

Mr. Alexander states that the Weisbach formula for the loss of head due to right-angle bends was derived from experiments by Weisbach on a 1-1/4 inch pipe.

His own tests were performed in 1901 and 1902 at the University of Birmingham.

Ten wooden bends, each 1-1/4 inches in diameter and with 6 inches of straight pipe on each side of bend, also an elbow. Two inches from each end of bend, 1/8 inch holes were drilled for piezometer tubes.

Difference of pressures read by gage using a mixture of chloroform and toluol. Velocities determined by weighing discharge.

Range of velocities, 1/4 to 8 feet per second. Curvature of bends from 0.1 to 0.500 in $\frac{r}{R}$, where r = radius of pipe, and R = radius of bend.

He finds the loss is the least when the radius of the bend is equal to $2\frac{1}{2}$ times the diameter of the pipe. He states "the second piezometer should have been farther away than 4 inches from the end of the bend, because that distance is insufficient for the disturbances set up by the bend to complete their effect, and consequently, the loss of head must have been greater than that given."

A set of equations are given which are supposed to give the loss of head due to bends.

I. P. Church.

Mechanics of Engineering. John Wiley & Sons, New York, 1902, 854 p., illus.

For pipe bends Church (p. 728) gives Weisbach's formula

$$K = 0.131 + 1.847 \left(\frac{A}{r}\right)^{7/2} \qquad \text{in which}$$

K = coefficient used in following formula

$$H = K \frac{V^2}{2g}$$

I.P. Church. (Cont'd)

a = radius of pipe
r = radius of bend to center line of pipe
V = velocity in pipe

Then for different values of $\frac{a}{r}$, he gives values of K.

For open channels he says "according to Humphreys' and Abbot's researches on the Mississippi River the loss of head due to a bend may be put

$$h_r = \frac{V^2}{536} \cdot \frac{6\alpha}{\pi}$$ in which

= velocity in feet per second

α = angle subtended by bend (must be in radians) Area F must be greater than 100 square feet and slope, s, must be less than .0008.

Mentions experiments of Prof. L.F. Bellinger on the loss of head caused by a common elbow (for wrought-iron pipe), made during his student days at Cornell, 1887. Velocity range 2 to 20 feet per second.

T. Claxton Fidler. Calculations in Hydraulic Engineering. Longmans Green & Co., 1902, Two Parts, Part 2, 203 p., illus.

Quotes Weisbach's formula for loss in bends and tabulates the coefficients to use.

Loss of head $h = \zeta_4 \frac{V^2}{2g}$

Weisbach proposes to determine the coefficient by the following formula

$$\zeta_4 = 0.9457 \left(\sin \frac{\theta}{2}\right)^2 + 2.047 \left(\sin \frac{\theta}{2}\right)^4$$

For circular bends of 90° with R = radius of curvature and r = radius of pipe, Weisbach gives the following values of ζ_4

Ratio $\frac{r}{R}$	0.1	0.2	0.3	0.4	0.5	0.6
Coefficients						
ζ_4	0.131	0.138	0.158	0.206	0.294	0.440

Fidler states these figures may suffice for a rough estimation of the loss of head due to bends but "further experiments are much to be desired."

Williams, Hubbell & Fenkell. Experiments at Detroit, Michigan, on the Effect of Curvature Upon the Flow of Water in Pipes. Trans. Amer. Soc. Civ. Engin. v. 47, 1902, pp. 1-335.

Experiments performed on cast iron water mains of Detroit, Michigan, from 1897-1901. About 21 experiments in all, 11 on 30-inch pipe, 2 on 16-inch and 6 on 12-inch pipe. all bends were 90-degree. Range of velocities from 0.5 to about 5.8 feet per second.

The radii varied by the following ratios:

30-inch pipe

$$\frac{\rho}{D} \quad \frac{\text{Radius of curvature}}{\text{diameter of pipe}} = 2.4,\ 4,\ 6,\ 10,\ 16,\ 24.$$

12-inch pipe

$$\frac{\rho}{D} = 1.08,\ 2,\ 3,\ 4.$$

16-inch pipe

$$\frac{\rho}{D} = 1.5,\ 2.26.\ 3.0,\ 3.76$$

The results showed that (1) "under some conditions, in straight pipe, there is a difference of pressure at different points around the circumference of the same cross-section"; (2) "The effects of disturbances of the flow of water in pipes are transferred for many diameters beyond the point where the interference occurs"; (3) "Curves of short radius, down to a limit of about 2½ diameters, offer less resistance to the flow of water than do those of longer radius, and hence that the theories and practices regarding curve resistance, as set forth in the hydraulic treatises of all nations up to the present time are absolutely incorrect and the diametric opposite of the true conditions."

In discussing this paper Irving P. Church points out the large error involved if the pipe is not exactly the same diameter at each of the sections where piezometers are made. Also that the resistance of flow was not increased because of the larger radius of bend but because of the longer distance around the bend.

J. L. Campbell also mentioned the latter point.

Williams, Hubbell & Fenkell. (Cont'd)	Charles W. Sherman condemned using one piezometer in the top of the pipe to record the pressure. Also stated long bends have many joints and, therefore, cause greater losses.
	D. Farrent Henry doubts ability to get air bubbles out of the water line. The long piezometer pipes of rubber are subject to constantly changing pressure as well as retaining air.
	H.W. Brickerhoff said that due to spiral motion the velocity increases and, as energy of moving body is proportional to square of the velocity, the energy absorbed will greatly increase. Also frictional resistance along edge of the pipe will increase. More resistance where spirals separate and come together again.
A.P. Folwell.	Notes on Lost Head in Water Supply Systems. Engin. News. v. 47, 1902, pp. 303-304.
	Quotes Weisbach's formula for loss of head due to curves and gives results of Williams, Hubbell & Fenkell tests.
J.C. Trautwine, Jr.	Resistance due to Bends in Pipes. Proc. Amer. Water Works Assoc., v. 24, 1904, p.328.
	Quotes Weisbach's formula and shows that by this formula resistance to flow in bend with right angle turn is greater than in a bend with a 90-degree curve.
Gardner S. Williams.	Proceedings Amer. Water Works Association. v. 24, 1904, p. 332.
	Mentions his own tests and shows equivalent losses in feet of straight pipe for various specials and curves in 30-inch water main.
E.C. Murphy.	Leliavski's Experiment on Stream Flow, Engin. News, v. 52, 1904, p.242.
	Discusses Leliavski's experiments on helical motion of water in streams and states that a section of a stream with such motion of filaments of water would not make a good gaging section.
L.M. Hoskins.	Hydraulics. Henry Holt & Co., 1906. p. 75.
	"The loss of head in a curved pipe is greater than that in an equal length of straight pipe.

L. M. Hoskins. (Cont'd)

For 90° curves of short radius, the formula usually given is that of Weisbach, who assumed

$$H' = K\frac{V^2}{2g}$$

and gave for K the empirical formula

$$K = 0.131 + 1.847\left(\frac{r}{R}\right)^{7/2}$$ in which r is the radius of the pipe and R, that of the bend.

"At all events, Weisbach's formula should not be used when the radius of the curve is greater than four or five times the diameter of the pipe."

Says experiments are too limited to derive general formula for longer radii.

R. Busquet.

A Manual of Hydraulics. Translated by A.H. Peake, Longmans, Green and Co., London, 1906, 312 p. illus.

"Pipe lines in different directions ought to be joined by easy bends...."

"It is evident that at the various changes in direction, the stream-lines collide with the sides of the conduit, and as a result shocks and eddies are produced causing losses of head which will be more important where the transition is least gradual, as where the junction is made by a sharp elbow, or a bend of small radius."

"It is impossible to determine these losses of head from theoretical considerations, but empirical formulae have been established, which enable an approximate estimation to be made for either sudden or gradual change of direction."

"The following formula established by Navier, is useful in calculating loss of head in bends:

$$P = \frac{u^2}{2g}\,(0.0127 - 0.0186\ r)\,\frac{a}{r^2}$$

where u = the mean velocity in feet per second
r = the mean radius of curvature of the bend in feet.
a = the length of arc of bend in feet."

A. W. Brightmore. Loss of Head in Water Flowing Through Straight and Curved Pipes. Proc. Inst. Civ. Engin. v. 169, 1907, pp. 315-336.

Experiments performed about 1902 at the (now non-existent) Royal Indian Engineering College, Coopers Hill. Brightmore made tests on loss of head in 3-inch, 4-inch, and 6-inch straight pipes 50 feet long, also loss of head at sudden contractions and enlargements and also loss of head due to bends.

Laboratory had a tank of 14,000 gallons. When full, its top water surface was 50 feet above laboratory floor. Cast iron volumetric measuring tank of same capacity placed beneath floor of laboratory.

Loss of head determined in three cast iron pipes, namely, 3-inch, 4-inch, and 6-inch diameter, each one 50 feet long. Friction measured in length of 30 feet, first piezometer 9 feet 3 inches from beginning, and fourth piezometer 8 feet 9 inches from end. Range of velocities, 0.20 to 1.17 feet per second and range of head, 0.02 to 1.95 inches per 20 foot length. Determined following equations for straight cast iron pipe, inside rusted (which was facilitated by sal ammoniac) but not tubercled.

3-inch pipe $V = 41.3 \sqrt{di}$

4-inch pipe $V = 47.5 \sqrt{di}$

6-inch pipe $V = 50.6 \sqrt{di}$

in which V = velocity in feet per second, d = diameter in feet of pipe, and i = loss of head per unit length.

Loss of head in bends on 3-inch and 4-inch pipe determined; 3-inch pipe tested in 90-degree elbow and 90-degree bends of radii equal to 2, 4, 6, 8, 10, 12 and 14 diameters; 4-inch pipe tested in 90-degree elbow and in 90-degree bends of radii equal to 2, 4, 6, 8 and 10 diameters. Inside of pipes uncoated and were allowed to rust.

"It was found that most of the loss of head due to the bend does not take place in the bend itself, but in the straight pipe following the bend." Consequently, in measuring the loss of head, a length of straight pipe sufficient for the flow to become normal again was included with the bend, and the normal resistance in a straight pipe equal in <u>length to the bend and this length of straight pipe was subtracted</u>, so that the losses plotted in the curves are the extra losses due to curvature." In the 3-inch pipe bend, the one piezometer was 4 inches preceding the bend and a

A.W. Brightmore.
(Cont'd)

piezometer 6 feet 8 inches following the bend, with a third 16 feet 8 inches downstream from the bend. In the 4-inch pipe bend, one piezometer was 7 inches upstream from the beginning of the bend, a second 5 feet downstream from the end of the bend, and a third 12 feet downstream from the bend. In some of the 4-inch pipe tests the second piezometer was 6 feet 7 1/4 inches downstream from the end of the bend.

"Since the loss of head is proportional to the square of the velocity, it may be inferred that the loss of head in bends in pipes of any diameter between 4 inches and 30 inches, if the radius be thus chosen, about 3 to 4 diameters will not be greater than that given for a 4-inch pipe bend of a radius equal to 4 diameters." "With bends of the most suitable radius the loss of head is independent of the size of the pipe." "It is also misleading to express the loss of head in inches per foot length of the bend, for, at least in the case of short bends, the excess resistance is developed in the length of pipe beyond the bend....."

"In order to ascertain how much of the loss of head actually took place in the bends and how much in the pipe followed the bend, due to the rearrangement of the velocities producing an excessive intensity of eddying motion there, it was necessary to measure the energy contained in unit weight of the water as it passed the end of the bend. For this purpose the distribution of pressure and velocity across the end-section of the bend had to be ascertained. In the first instance, the difference of pressure at the extremities of the horizontal diameter of that section was measured for the five bends in pipes of 4 inches in diameter. It will be seen that the pressure was always greatest at the outside of the curve, and that the difference of pressure in some cases was considerable. This suggested that the water moved round the bend more or less in a free vortex, i. e., that the energy in each unit weight of water remains constant except for the loss of pressure due to resistance. In order to be able to compare the theoretical values of the difference of pressure at the inside and outside of the bend, on this assumption, with the actual measurements, it is necessary to deduce an expression for that difference."

"The formula $$\frac{p_o - p_i}{g} = \frac{V^2_{mean}}{2g} \times 4 \frac{R_o - R_i}{R_o + R_i}$$

may be used to compute the difference of head between the outside and inside of the bend."

A.W. Brightmore.
(Cont'd)

"The energy due to the velocity will be about the same at the commencement, because, although the velocity is not uniform across the sections, its mean value is of course the same. Taking, therefore, the energy due to velocity to be the same at the end of the bend as at the beginning, the loss of head in the bend will be equal to the difference of the pressure head at the two ends of the bend. In order to estimate the pressure head in the water at the end of the bend, it is necessary to know the value of the mean pressure, p; p must therefore be found in terms of p_o and p_i."

$$\frac{p}{g} = \frac{p_o}{g} - \frac{p_o - p_i}{g} \times \frac{R_i}{R_i + R_o} \qquad (5)$$

"Formula 5 enables the pressure head at the end of the bend to be calculated when the pressure at the outside and the difference between the pressures at the outside and inside of the bend are known."

"It will also be noticed that the losses of head in the bends themselves are not generally greater than the loss in an equal length of straight pipe for the same velocity, and therefore the additional loss of head due to the presence of the bends must take place in the redistribution of the velocities in the pipes following them."

Brightmore then makes velocity traverses across the end section of a 4-inch pipe bend on its horizontal diameter. He gets higher velocities on the inside of the end of the bend than on the outside at the end of the bend. He does not speak of the dead space near the inside wall at the end of the bend which is so apparent in the Iowa tests.

Brightmore's conclusions follow in part:

(4) "That the additional loss of head in right-angled bend pipes of these diameters has a minimum value for bends of a radius equal to four diameters, irrespective of the velocity of flow."

(5) "That the additional loss of head in pipes of these diameters attains a maximum for a radius of bend of six or seven diameters and falls again for bends of greater radii."

(6) "That the loss of head in right-angled bends of a radius giving this minimum loss is independent of the size of the pipe, and depends only on the velocity of flow and the condition of the internal surface of the pipes."

A.W. Brightmore. (Cont'd)

(7) "That the additional loss of head in the case of short bends occurs in the length of pipe following the bend, owing to the intensified eddying-motion there, due to the rearrangement of velocities."

(8) "That in the bends experimented upon, the flow tends to approximate to a free vortex, the pressure being greatest at the outside and least at the inside of the bend."

(9) "The last conclusion is confirmed by the fact that the velocity was found to be a maximum at the inside of the bend and a minimum at the outside, their relative values being approximately inversely proportional to the radii."

W.C. Unwin. A Treatise on Hydraulics. Adam and Charles Black, London, 1907, 324p. illus.

Discusses resistance at pipe bends. Gives Weisbach's formula, Alexander's, and Williams', Hubbell and Fenkell's tests. Discusses flow in river bends and gives Prof. Thomson's theory and experiments in detail.

W.B. Gregory and E.W. Schoder. Some Pitot Tube Studies. Trans. Amer. Soc. Mech. Engin. v. 30, 1908, p. 351.

Shows some velocity traverses made by Pitot tube at various sections in a bend.

Gardner D. Hiscox. Hydraulic Engineering. Norman W. Henley Pub. Co., New York, 1908, 315 p. illus.

"The loss of head by long bends, say of a radius of ten diameters or more, is very small and practically needs no consideration."

Mentions Weisbach's formula.

H. T. Bovey. Hydraulics. John Wiley & Sons, New York, Ed. 2, 1909, 585 p. illus.

Quotes Weisbach's and Navier's formulas, and mentions William's, Hubbell and Fenkell's tests. In flow around river bends he gives Prof. James Thomson's theory and experiments together with his diagrams.

F.C. Lea. Hydraulics. Edward Arnold, New York, 1908, Second Impression, 536 p. illus.

"Weisbach (Mechanics and Engineering) from experiments on a pipe 1 1/4 inches diameter, with bends of various radii, expressed the loss of head as

F.C. Lea. (Cont'd)

$$h_b = (0.065 + \frac{0.923r}{R}) \frac{V^2}{2g}$$

r being the radius of the pipe, R the radius of the bend on the center line, and v the velocity in feet per second. If the formula be written in the form

$$h_b = \alpha \frac{V^2}{2g}$$

the table shows the values and α for different values of $\frac{r}{R}$."

$\frac{r}{R}$	α
0.1	0.157
.2	.250
.5	.526

"St. Venant (Comptes Rendus, 1862) has given as the loss of head h_B at a bend,

$$h_b = 0.00152 \frac{1}{R}\sqrt{\frac{d}{R}} V^2 = 0.1 \frac{V^2}{2g} \frac{1}{R}\sqrt{\frac{d}{R}} \text{ nearly.}$$

1 being the length of the bend measured on the center line of the bend and d the diameter of the pipe.

When the bend is a right angle

$$\frac{1}{R}\sqrt{\frac{d}{R}} \quad \frac{\pi}{2}\sqrt{\frac{d}{R}}$$

When

$$\frac{d}{R} = 1 \qquad 0.5 \qquad 0.2$$

$$h_b = .157\frac{V^2}{2g} \qquad .111\frac{V^2}{2g} \qquad .07\frac{V^2}{2g}$$

He also gives Alexander's formulas.

Lea then (page 520) discusses Thomson's theory of flow around bends in open channels.

E.W. Schoder.

Curve Resistance in Water Pipes. Trans. Amer. Soc. Civ. Engin. v. 62, 1909, pp. 67-112.

In 1907, E.W. Schoder made at the Cornell University hydraulic laboratory, experiments on 6-inch, 90-degree pipe bends of 12 different radii (0.67 ft. to 10.0 ft.). Velocities obtained by a calibrated brass nozzle. Pipe used was wrought iron and cast iron. Range of velocities from 2.5 to 16.6 feet per second. Two piezometers were used to get loss of head; one, 1.10 feet upstream

E.W. Schoder. (Cont'd)

from the bend and one 84.1 feet downstream from the bend. Two piezometers connected to a U-tube mercury manometer by means of 1/2-inch rubber hose.

Schoder concludes that his experiments together with Brightmore's show a decreasing loss of head for an increasing radius of curvature.

In connection with these same tests Schoder and Gregory made, by means of a Pitot tube, velocity studies at points 22 1/2 degrees, 45 degrees and 67 1/2 degrees from the upstream end of the curve with the 2.5 foot radius. Traverses were made both in the vertical and horizontal diameters. These traverses (See "Some Pitot Tube Studies", by W.B. Gregory and E.W. Schoder, Proc. Amer. Soc. of Mech. Engrs., v. 30, 1908, p. 351) show quite similar results as obtained in the Iowa tests.

E.S. Bellasis.

Hydraulics with Working Tables. Spon and Chamberlain, New York, Ed. 2, 1911, 311 p. illus.

"For bends in small pipes Weisbach found the loss of head to decrease as the radius of the bend, R, increases. Later experiments by Brightmore, Schoder and Davis, on pipes of diameter ranging from 2 to 6 inches, show that H_B is a minimum when R is about 3.5D, increases somewhat as R increases to about 7D, decreases till R is about 15D, and then increases as R increases from 15D to 20D.

Bellasis then gives a table of coefficients, K, in the formula

$$H_B = K \frac{V^2}{2g}$$

taken from the experiments of Weisbach, Davis, Brightmore, Schoder, and williams, Hubbell and Fenkell.

Coefficients of Schoder are smaller in every case for small pipes, below 6-inch, than are those of other experimenters.

For open channel bends Bellasis gives the formula

$$H_B = \frac{V^2 \sin^2 \theta}{134}$$

which reduces to $0.48 \frac{V^2}{2g}$ for a 90-degree bend

E.S. Bellasis .(cont'd)

He then quotes James Thomson's theory and experiments.

"As the transverse surface slope cannot commence or end abruptly there is a certain length in which they vary. In this length the radius of curvature of the bend and the form of the cross-section also tend to vary. This can often be seen in plans of river-bends, the curvature being less sharp towards the ends. This principle has been adopted in constructing river training walls, and it appears to be sound as tending against any abruptness in the change of section. For training-walls to remove bars at the mouth of the Mississippi it has been proposed to construct, instead of two walls, only one wall having a curve concave to the stream." Ed. 4. page 249.

"At a bend there is a 'set of the stream' towards the concave bank, the greatest velocity being near that bank; and there is a raising of the water level there, so that the surface has a transverse slope. There is also a deepening near the concave bank and a shoaling at the opposite one but this is not all due to the direct action of centrifugal force."

G.R. Davis. Investigation of Hydraulic Curve Resistance. Experiments with two-inch pipe. Bulletin No. 403. Univ. of Wis., Madison, January, 1911. 60 p.

In the Wisconsin experiments by Davis, piezometers downstream from the bend were placed 1 foot, 7 feet, and 17 feet or 5.82, 40.7 and 98.9 diameters downstream.

Tests were made on tees and 90-degree elbows in 1908 and 1909 at the University hydraulic laboratory. From 12 to 26 tests were made on 12 different 90-degree bends, all 2 inches in diameter. Range of velocities 1.6 to 14.4 feet per second. Radii of bends ranged from 0.135 feet to 7.08 feet. Velocities measured by volumetric basin. Pipe used were cast iron, wrought iron and malleable iron.

Net loss curve plotted showing variation in loss with radius of pipe.

John Eustice. Flow of Water in Curved Pipes. Proc. of the Royal Society of London, Series A, v. 84, Feb. 1911, pages 107-118. (Communicated by Sir J. Lamor, Secy., R. S. Received April 25, - Read June 2, 1910.)

Tests conducted at Hartley University College. Apparatus consisted of a supply tank 2 feet in diameter giving a direct head of 36 feet (1,100 centimeters). Supply tank connected to town water mains. Rubber tubing coiled ten times around wooden cylinders used for tests. Area of tubing 0.0735 square centimeters determined volumetrically.

John Eustice. (Cont'd)

Gages to determine loss of head set 10 centimeters apart. Total length of tube between gages 97.8 centimeters. Length of coiled part of tube 81.5 centimeters. Rubber tubing coiled around wooden cylinders which were turned to the diameters necessary to give a whole number of coils for a given length of tube. Amount of flow measured volumetrically. Time of one test about 100 seconds. Six different coils tested; about 20 tests for each coil. Range of loss of head measured was from 0.53 to 1082.0 centimeters. Tests also made on rubber tubing stretched out full length placed on both level and inclined positions and on tubing squeezed into oval-shaped cross-section.

Summary of results:

1. "The flow in straight flexible tubes obeys the laws of flow in metal tubes as investigated by Prof. Osborne Reynolds (Phil. Trans. 1883)."

2. "The flow obeys the index law both below and above the critical velocity, that is, in plotting logarithmically head against discharge the lines below and above the critical velocity are both straight."

3. "The increased resistance expressed in terms of the loss in quantity discharged, or in loss of velocity for a given loss of pressure, is given approximately by the formula:

$$\frac{dQ}{Q}^{n} = \frac{dV}{V}^{n} = CR^{-1}$$

In which R is the radius of curvature of the coil.
C is a constant for a constant hydraulic gradient.
n is index of V in the formula $S = \frac{KV^n}{m}$

4. "The slow recovery of the flexible tube from the strain due to deformation caused by squeezing or by coiling adversely affected the experiments; but the results of the time lag were not sufficiently serious to affect materially the general deductions which have been made."

Experiments on Stream Line Motion in Curved Pipes. Proc. Royal Society of London, v. 85, pp. 119-131. Paper received January 26, - Read February 16, 1911.

Prof. Eustice studied the motion of water in both U-shaped or 180-degree bends and 90-degree bends. The 180-degree bends were circular glass tubes, 1 centimeter in diameter and of the following radii 2, 4, 6, 8, 10, 25, 50 and 100 centimeters. The 90-degree bends were glass tubes, a centimeter in diameter and of the following radii, 2, 4, 6, 8, and 10 centimeters; also made tests in a 360-degree tube and abrupt or sharp right angled bends.

John Eustice. (Cont'd)

Velocities ranged from about 3 centimeters per second up to 18 centimeters per second. One test made on a 180-degree bend with a diameter of 1.7 centimeters, all other diameters 1 centimeter.

Motion of water studied by introducing various colored liquids. Color introduced through separate nipples. Outlet end of tube connected to a rubber tube on which a petcock was placed to regulate the quantity of flow. All velocities tested were below critical velocity by Reynold's formula.

"Thomson's experiments were carried out in open channels, and he has suggested that the theory of flow in curved channels is applicable to pipe bends. In introducing the theory he states that a stream flowing along a straight channel, and thence into a curve, must flow with a diminished velocity along the outer bank and an increased velocity along the inner bank. The author's experiments show that the motion at the commencement of a curve of a pipe bend is not that of a free vortex, for the velocity is greater near the outside than it is near the inside of the curve."

"There is another difference between the two cases. In an open channel there is freedom in a vertical direction, and, as Thomson has pointed out, the difference of pressure between the inner and the outer banks causes the surface of the water to be inclined from the inner to the outer bank of the curve. No such freedom is possible in a pipe which is running full, hence it is probable that the curvature of the stream lines is greater and the effect of pressure on the transfer of water from the outer to the inner curve is more pronounced in a pipe than in a channel."

"The surface flow which commenced in the bend generated vortices which persisted in the water flowing through the outlet straight part of the pipe. This shows that the effect of a pipe bend or an angle is not only to increase the resistance to flow in the bend itself, but also to increase the resistance in the contiguous straight pipe after the water has left the bend."

Anonymous.

Loss of Head in Screw Pipe Elbows and Tees. The Cornell Civil Engineer. v. 20, 1911. pp. 107-113.

Discusses tests of Daley, Schoder, Bain, and Davis. Gives values of K in formula

$$H = K \frac{V^2}{2g}$$

for various tests.

E.G. Hopson. Engineering Record. v. 64. 1911. pp. 480-481.

Measured losses due to friction and curvature in concrete lined canal on Umatilla project in Oregon. Found Kutter's n in curve much higher than on tangent.

Philip J. Markmann. A Theoretical Formula for the Curve Resistance to the Flow of Liquids. Paper before the St. Louis Engineers Club, No. 2, 1911; Canadian Engineer, v.21, pp. 504-507.

Quotes Williams, Hubbell and Fenkell tests, also Brightmore's, Schoder's and Freeman's experiments; Derives a new formula

$$H_2 = \frac{lcv^2}{2gr^2}$$

in which l = length of curve in feet, r = radius of curve in feet and H_2 = loss of head for entire curve.

I. Isaachsen. Innere Vorgange in stromenden Flussingkeiten und Gasen. Zeitschrift des Verainces deutscher Ingenieure, Band 55, February and March, 1911.

"The method of approaching problems of flowing water by way of the so-called classical hydraulics (assuming flow without friction) may lead to serious misunderstanding of the actual nature of flow in reality. The friction in flow cannot be represented by a single coefficient applied to the theoretical results obtained by the above mentioned method of reasoning.

"Secondary currents, the result of friction on flowing fluids, play a far more important part in the behavoir of fluid motion than most people realize. These secondary currents are always created when the main flow is diverted."

Discusses various cases where secondary currents exist. The spiral flow in closed bends, the vortices produced in the water turbine runner, the vortices occasioned by the slight sideways motion of a ship, and the condition of unequal pressure and velocity in a quarter turn draft tube are all mentioned and discussed.

E.S. Bellasis. River and Canal Engineering. Spon & Chamberlain, London and New York, 1913, 215 p. illus.

Bellasis discusses the action which takes place at bends. "When once a stream has assumed a curved form, be it ever so slight, the tendency is for the bend to increase. The greater velocity and greater depth near the concave bank react on each other, each inducing the

E.S. Bellasis. (Cont'd)

other. The concave bank is worn away, or becoming vertical by erosion near the bed, cracks, falls in, and is washed away, a deposit of silt occurring at the convex bank, so that the width of the stream remains tolerably constant. The bend may go on increasing, and it often tends to move downstream."

Anonymous. Engineering Record, v. 68, 1913. pp. 381-382.

Article discusses various tests made on pipe bends and gives loss of head for bend with radius of bend expressed both in feet and in diameters of pipe.

———— Engineering Record. v. 68, 1913, pp. 727-728.

A new formula is deduced based on experiments as well as theory. Other formulas are discussed briefly.

Loliger, Untersuchung des Druck und Strömungsverlaufs in Schaufeln, Dissertation, Zurich, 1913. Discusses spiral flow in bends.

Leland Rolla Balch. Investigation of Hydraulic Curve Resistance. Experiments with Three Inch Pipe. Bull. 578, Univ. of Wis. Madison, 1913, 51 p.

All bends tested were 90-degrees with radii varying from 0.182 to 5.00 feet and with values of R/d ranging from 1 to 20. Velocities ranged from 4 to 10 feet per second. One piezometer was placed 1 foot above and one 9 feet below the bend.

The following conclusions were drawn by Mr. Balch:

1. The total loss of head in bends decreases with an increase in radius until the radius equals about 4 pipe diameters. For bends with radii greater than 5 pipe diameters the total loss increases with an increase in the length of radius.

2. The curve factor, f, in the equation $h = f \frac{LV^2}{d\,2g}$ is independent of the diameter of the pipe but varies inversely with the length of the radius. There is apparently a critical value of the radius of bend at about 5 pipe diameters below which the rate of change in the value of f is much greater than for values of R/d above 5.

3. The net curve resistance or loss of head due to the bend alone, not including pipe friction, decreases with an increase in radius of curvature to a value of R equal to about 6 pipe diameters, and increases thereafter until the radius is equal to about 14 pipe diameters after which it decreases.

Leland Rolla Balch.
(Cont'd)

4. The net curve resistance per unit length of bend is independent of the diameter of the pipe. It decreases with an increase in the radius of curvature and varies approximately with the square of the velocity of flow. There is seemingly a critical value of the radius of the bend at about 5-1/2 pipe diameters, below which the rate of change in the net loss per unit length with respect to the radius of curvature is much greater than for values of radius larger than 5-1/2 diameters. The net curve resistance as given by the following two formulas appears to give fair estimate of the loss in bends having radii between 1 and 15 pipe diameters.

For values of R/d less than 5-1/2

$$\text{Loss } h = 0.0043\ (R/d)^{-1.546}\ V^{2.079}$$

For values of R/d between 5-1/2 and 18

$$h = 0.0004\ (R/d)^{-0.184}\ V^{2.079}$$

W.E. Fuller.

Loss of Head in Bends. Jour. New England Waterworks Assoc. v. 27, 1913, pp. 509-521.

Fuller states that experiments made by Schoder, Davis, Brightmore and others show that the Weisbach formula does not hold for larger pipes under ordinary conditions of service. "It seems to have been assumed that the loss of head in bends on different sizes of pipe should be the same when the radius of the bend in terms of the diameter of the pipe were alike."Fuller sees no reason why this should be so. He plots one set of curves using all past experiments, with loss of head due to the bend as ordinate and the radius of the bend in terms of diameters as the abscissa. Another set is plotted using "radius of bend in feet" as the abscissa and the same ordinate as before. These last curves appear quite consistent.

Fuller then plots for a 5 foot velocity for different size of pipe, the excess loss of head in 90-degree bends over that in tangents using radius of bend in feet as abscissae. He suggests certain values for loss in 90-degree bends. Then for 45-degree bends, he suggests using 3/4 that due to 90-degree bend of same radius. For 22.5-degree bends use 1/2 that due to 90-degree bends of the same radius.

R.H. Gockinga.

The Transverse Slope and Its Influence Upon the State of Rivers. Annales des Pontes et Chaussees, v. 83, (1-3) pp. 112-133, 1913.

R. H. Gockinga. (Cont'd)

Gockinga discusses the flow of water around bends and derives a formula for computing the difference of elevation of the water surface transversely across the bend. His formula is

$$y = 0.235\ V^2\ \log\ \left(1 + \frac{X}{R}\right)$$

in which
R = radius of the bend to axis of stream
x = distance to filament to be considered
v = velocity of filament (mean velocity if total width is considered)
y = difference of elevation of water surface.

The difference of level between the two banks is obtained by substituting successively + b/2 and -b/2 for x and by subtracting one from the other the values of y thus obtained.

Gockinga discusses spiral flow and mentions that M. Boussinesq in his "Essay on the Theory of Running Water" Paris, 1877, says a "quasi-helicoidal movement" must take place if there is a difference of elevation of the water surface transversely.

W. Henry Hunter. Rivers and Estuaries or Streams and Tides. Longmans Green and Co., 1913, 69 p. illus.

A discourse is given on the flow of water around bends. Quotes James Thomson's experiments. Hunter says "the water flows more swiftly" beneath the outer bank and "its erosive effect is greater than on the opposite side."

Philip A. Morley Parker. The Control of Water, D. Van Nostrand Co., New York, 1915, 1037 p. illus.

A review of Saph and Schoder's experiments and of Brightmore's tests is given. Their tests "show very clearly that curve resistance is caused by a redistribution of the velocities over the cross-section of the pipe."

Parker then gives Alexander's formulas, also a diagram showing Brightmore's and Davis' results plotted with loss of head in feet as the ordinate and the ratio of the radius ofthe bend to the diameter of the pipe as the abscissa. He also gives Weisbach's formulas and states that Bellasis in The Engineer, May 26, 1911, discussed all recorded experiments on loss of head at bends in pipes. He compares in a table the value of the coefficient in the formula

$$h_a = (\zeta_a)\ \frac{V^2}{2g}$$

for the various tests and finds they vary widely. He says the values are most unreliable.

Frank L. Busey.

Loss of Pressure Due to Elbows in the Transmission of Air through Pipes or Ducts. Amer. Soc. of Heat & Vent. Engineers Trans. 1913, p. 336.

Dr. Jacob Lell. Beitrag zur Kenntnis der Sekundarstromungen in gekrummten Kanalen. Zeitschrift fur das Gesamte Turbinenwesen Heft 19, 20, and 31, July 10, 1914, pp. 293-298, July 20, 1914, pp. 313-317, July 30, 1914, pp. 325-330.

Conducted tests on 180-degree bend of rectangular cross-section, 200 mm (7.87 inches) wide and 100 mm. (3.94 inches) deep with 100 mm. inner radius. Approach tangent 3131 mm. long (123.23 inches). Discharge tangent 2,000 mm. long (78.74 inches). At numerous cross-sections in the bend, ten piezometers were placed on bottom of channel, five on each side and from four to eleven on top.

Sides and top of bend and of 1,000 mm. (39.37 inches) of approach tangent next to bend and entire length of discharge tangent made of transparent material, (glass).

Tests made in three quantities giving velocities of 3, 6, and 9 meter per second. The heights of the water surfaces in the piezometers tubes for 18 transverse sections were photographed.

R. De Villamil. Motion of Liquids. E & F.N. Spon, Ltd., London 1914, 210 p. illus.

Mentions Prof. James Thomson's work and quotes part of Thomson's paper in Proceedings Royal Society 1876. He states that Thomson says the highest velocity is along the inner bank. De Villamil then says that the river Indus did not flow like that. "The maximum velocity of the stream (in the river Indus) was always nearer the outer than the inner bank."

"I am aware of what James Thomson calls the Hydraulic Principle, and I quite agree that, if the liquid had no free surface, the velocity near the inner bank would be in excess of that near the outer bank - whether the liquid were viscous or not. Water flows like that in a pipe." States that Bellasis also considers the greatest velocity next to the concave bank.

Dr. Hubert Engles. Handbuch des Wasserbaues. Leipzig and Berlin, Verlag Von Wilhelm Englemann, 1914, Part One, pp. 118-119 and 323-325.

Gives the following formula for bends in pipes:

$$h_1 = \int \frac{V^2}{2g} \frac{\alpha}{90^\circ}$$

Dr. Hubert Engles. (Cont'd)

in which $\zeta = 0.131 + 1.847 \left(\frac{r}{R}\right)^{7/2}$

Give Weisbach's values for ζ for different values of $\frac{r}{R}$.

For flow in bends of open channels, he mentions the spiral motion of the water in the bend and gives the following formula for determining the transverse slope of the water in bends

$$\operatorname{Sin} \alpha = \frac{V^2}{g^r}$$

in which sin α is the transverse slope of the water surface at the bend and r is the radius of the bend.

Mansfield Merriman. Hydraulics. Ed. 9. John W. Wiley & Son, N.Y. 1914. 565 p. illus.

Merriman adopts a formula of the form $h_B = f_1 \frac{l}{d} \frac{V^2}{2g}$

where l = length of curve.
d = diameter of pipe
v = mean velocity of flow
f_1 = an abstract number

Then for al l the various experimenters; Weisbach; Castel; Freeman; Williams, Hubbell and Fenkel; Schoder; Davis; Brightmore, he gives for various ratios of

$$\frac{R}{d} = \frac{\text{radius of bend}}{\text{diameter of pipe}}$$

values for the coefficient f_1.

He also discusses Thomson's theory.

E.H. Sprague. Hydraulics. Scott, Greenwood & Son, London, 1914. p. 184 illus.

Mentions resistance to flow of pipe bends. Gives values of C_r in the formula $H_1 = C_R \frac{V^2}{2g}$

for various ratios of diameter of pipe to radius of bend.

W.M. Wallace. Hydraulics for Engineering Students and Engineers in Practice. The Technical Publishing Co., London, 1914, 280 p. illus.

W.M. Wallace. (Cont'd)

"At a bend in a river the centrifugal action tends to lift the surface of the water near the bank further removed from the center of curvature, and the head of water above the bed here is greater than that at the opposite bank; consequently, currents are set up crossing the bed from the place of higher pressure to the lower. The velocity of the stream is higher at the outer bank and the turbulent motion due to this, as well as the impact of the water on the outer bank, breaks up the sides and the particles of sand and gravel are carried across the stream and deposited by the inner bank. This accounts for the great changes which have obviously occurred in the course taken by streams."

Louis A Martin. Textbook of Mechanics. v. 5, Hydraulics. John Wiley & Sons, New York. 1914. 223 p., illus.

"No theory has been devised which leads to even approximate values for these losses (elbows and bends, valves and gates.) Experimental investigations of these losses are few in number and they are usually made on pipes of small size. Fortunately on long pipe lines the frictional resistances are so much larger than the secondary resistances that these need seldom to be taken into account."

Fred C. Scobey. The Flow of Water in Irrigation Channels. U.S. Department of Agriculture. Bull. 194, 1915. 68p. illus.

Recommends a slightly higher value of Kutter's n for channels having sharp curves than for straight channels.

Frank S. Bailey. Diagram giving excess loss in 90-degree bends. Engin. News, v. 75, 1916, pp. 412-413.

A chart giving excess loss of head in 90-degree bends in cast-iron water pipe is shown. This chart is based on the formula

$$H_B = K V^{2.25}$$

in which K is a coefficient, different for bends of different radii.

Ben Morrell. Lost Head Diagrams for Bends in Water Pipe. Engin. News, v. 75, pp. 302-4.

For velocities of 3, 5, 7.5, 10 and 16 feet per second, Morrell plots on a separate diagram for each of these velocities all experimental data with lost head in feet as ordinate and the ratio of the radius of the curve to the diameter of the pipe R/d as abscissa. Then he plots by interpolation, on a diagram using the same ordinates the losses for various velocities from 3 to 16 feet per second.

Ben Morrell. (Cont'd)

He leaves out the Williams, Hubbell and Fenkell data because he considers the piezometers were placed too close to the beginning and end of the bends.

Morrell's curves for lost head are for 90-degree bends. He suggests that for 135-degree bends allow 4/3 loss in 90-degree bends of same radius. For 45-degree bends, allow 2/3 loss in 90-degree bend of same radius. For 22-1/2-degree bends, allow 1/2 loss in 90-degree bend of same radius. Morrell considers the loss the same regardless of the size of the pipe although he says it is not correct to do so but it will have to be done until more experiments are made on large pipes.

F. zur Nedden. Induced Currents of Fluids. Trans. Amer. Soc. Civil Engin. v. 80, 1916, pp. 844-896.

Discusses secondary currents in pipes and pipe bends. States that according to Isaachen's experiments the velocities of stream lines are inversely proportional to their radii of curvature. If r_a and r_i represent the outer and inner radius, respectively and v_a and v_i the corresponding velocities at these radii, the following equation holds good:

$$V / v_i = r_i / r_a$$

or $v_a \; r_a = v_i \; r_i = v_m \; r_m = C$, where C is a constant, and r_m the radius of the neutral axis, and v_m the velocity at this axis which is equal to the mean velocity in a straight channel of identical cross-section.

"The critical speed for water at the same temperature, 65 to 70-degree Fahrenheit, and ordinary 4-inch iron pipe, is about 1 foot per second."

The difference of the outside and the inside pressures in the bend $v_1^2 \quad 2g \quad -v_a^2 / 2g$

It was found that the induced currents originate in the strata nearest the covers (of Loll's apparatus) and gradually increase in volume and intensity while the main current flows foward. States that the extra resistance caused by a circular pipe or coil as observed by Weisbach does not exceed that of its first quadrant. This fact further permits us to apply to 90-degree bends Pfarr's results on a 180-degree bend.

F. zur Nedden. (Cont'd)

The extra loss observed in bends comes from: (1) "The loss of head caused by impact of the induced currents against each other midway between the top and bottom. (2) The resistance in free cross-sectional area owing to the space taken up by eddies accompanying this impact, (3) The losses of head due to repeated acceleration and retardation of every single vein changing its speed in rapid succession."

The roughness of the walls of the bend can only have a subordinate influence on the resistance due to curvature.

A considerable part of the article is devoted to the effect of bends in suction pipes on the efficiency of centrifugal pumps.

Discussion of paper by Clemens Herschel, Carl George De Laval, and John Trautwine, Jr.

James Park. Loss of Head Due to Bends in Circular Pipes. A text book of Practical Hydraulics, Chas. Griffin & Co., London, 1916, 284 p. illus.

"Weisbach's formula $h_1 = \zeta \dfrac{\theta}{180°} \quad \dfrac{v^2}{2g}$

in which h_1 = loss of head in feet

θ = angle through which pipe is bent
ζ = a coefficient of resistance

"The value of ζ is deduced from the empirical formula

$$\zeta = 0.131 + 1.847 \left(\frac{r}{R}\right)^{7/2}$$

in which r = radius of the pipe itself
R = radius of the curve to the center line of pipe.

Values for ζ are given for various values of $\frac{r}{R}$.

Mentions in curves in flumes that "the water has a tendency to pile up against the concave bank."

Fred C. Scobey. The Flow of Water in Wood-Stave Pipe. U.S. Department of Agriculture. Bull. 376, 1916, 96 p. illus.

F.E. Giesecke. The Friction of Water in Iron Pipes and Elbows. Amer. Soc. Heat. & Vent. Engin. Jour. July 1917.

Gives experimental results.

S.E. Slocum. Elements of Hydraulics. McGraw Hill Book Co., New York. Ed. 2, 1917, 329 p. illus.

Quotes Weisbach's formula, also William's. Brightmore's, Davis', Balch's, and Fuller's work are mentioned. Gives Fuller's curve, table and conclusions in much detail.

H.W. King. Handbook of Hydraulics. McGraw-Hill Book Co., New York, 1918. 422 p. illus.

King gives W.E. Fuller's formula $H_b = CV^{2.25}$

in which h_b = loss in head for 90-degree bends; c, is a coefficient varying with the radius of the center line of the pipe. He gives Fuller's values for c.

King then gives the formula $H_b = K \frac{V^2}{2g}$

and gives values for K (page 186). He quotes Fuller's conclusions.

Edward Wegmann. Conveyance and Distribution of Water for Water Supply. D. Van Nostrand Co., New York, 1918, 663 p. illus.

"The first experiments on the additional loss of head caused in a water pipe by curves were made, about 1777, by Bossut (See The Foundation of our Knowledge of Hydraulic Curve Resistance by G.S. Williams, Michigan Technic, 1899, p. 48), with a pipe 0.09 feet in diameter and about 53 feet long. DuBuat (1786), Venturi (1797), and Rennie (1831) also made experiments on the resistance caused by curves in pipes."

Wegmann gives Weisbach's coefficients, also J.R. Freeman's tests on fire hose, Williams' Schoder's, Brightmore's and Davis' tests. He gives as a conclusion the curves drawn by Ben Morrell, (Engin. News, Feb. 17, 1916, page 302) Ass't Engineer on the Water Works of St. Louis. Morrell does not include the Detroit tests in his curves. Wegmann advises using Morrell's curves to determine losses for various velocities and ratios of $\frac{R}{D} = \frac{\text{Radius of Curve}}{\text{Radius of Bend}}$.

"The conclusion of Williams, Hubbell, and Fenkell 'that curves of short radius, down to a limit of about two and one-half diameters, offer less resistance to the flow of water than do those of longer radius' is not in accordance with the careful experiments made subsequently by A.V. Saph and E.W. Schoder, George J. Davis, Jr., and Arthur W. Brightmore."

Frank S. Bailey. Diagrams for excess loss of head in pipe lines. Engin. News-Record, v. 83, 1919, pp. 162-163.

Gives three charts for determining loss of head in 90-degree bends, gate valves, and increasers based principally on Fuller's formula. Questions reliability of the theory on which the Markmann formula is based.

Dean E. Foster. Effect of fittings on flow of fluids through pipe lines. Amer. Soc. Mech. Engin., v. 42, 1920, pp. 647-669.

Abstracted in Engin & Constr., v. 54, 1920. p. 564.
" in Mech. Engin., v. 42, 1920, pp. 616-618.
" in Power Plant, v. 24, 1920, pp.115-118.
" in Power, v. 52, 1920, pp. 1032-1033.

Quotes Meier's formula for determining resistance offered to flow by fittings and gives table to show equivalent lengths of straight pipe to allow for various screw fittings. In the discussion on the paper, F.E. Giesecke, page 661-662, questions the accuracy of Meier's formula and mentions tests made by himself on pipe fittings in 1911 and 1912 at the Texas A & M. College and again in 1916 and 1917 at the University of Texas, later published in University of Texas Bulletin No. 1759, "The Friction of Water in Pipes and Fittings."

Meier's points out, in discussion, that a formula is merely an approximation for convenience.

Mansfield Merriman. American Civil Engineers Pocket Book. John Wiley and Sons, New York, 1920. Ed. 4. 1591 p. illus.

"The effect of curvature (in pipes) is to increase the loss of head. The loss of head due to a curve may be stated in terms of the velocity head h_v, or, better, in terms of the equivalent length of straight pipe which would give the same loss as the curve."

"The loss in the curve should be measured as the excess over that in an equal length of straight pipe." He then quotes W.E. Fuller's conclusions as given in the New England Water Works Association, Dec. 1913.

Mansfield Merriman. (Cont'd)

On page 1717 (Sec. 16, Harbor and River Works, by Admiral F.R. Harris) a discussion of erosion and scour at river bends is given. Thomson's theory is not mentioned but diagrams similar to his are shown and the same theory of spiral flow is given.

R.C.R. Minikin.

Flow Round Bends. Practical River and Canal Engineering. Chas. Griffin & Co., London, 1920, 119 p. illus.

"The least understood of all river phenomena is that of the flow round bends and the action of the water in its passage."

"The general facts are:--

1. There is an increasing deposit on the inner or convex bank.
2. There is constant erosion on the outer or concave bank.
3. That the radius of curvature tends to decrease.
4. That the greatest depth is under the concave bank."

Minikin then gives Thomson's theory and his diagram. He states the deepest part of the river is downstream from the middle of the bend near the outer bank. Likewise the highest deposit is downstream from the middle of the bend and is next to the inner bank.

"The line of maximum velocity depends upon the curvature." "The deposit does not take place until the direction of the current tends to leave the inner bank and finishes where the general direction of the current again meets it, after being deflected by the outer bank."

"The greatest depth is always in proximity to the greatest resistance to the flow of the water, and is due to the increased local disturbance consequent on the waters being deflected from their original path."

"As the greatest erosion is always downstream of the summit of the curve, the tendency is for the bend to travel downstream."

Gives notes on actual cases."....facts empahsize the weakness of theorizing on the flow of rivers, and the engineer who tackles river problems for the first time must study the particular case with greatest care, and find his own facts."

S. M. Woodward. Hydraulics of the Miami Flood Control Project. Technical Report, Part VII, Miami Conservancy District, Dayton, Ohio, 1920, 343 p. illus.

"The layers (of water) at the bottom have a much slower velocity than the central layers or top layers." "The fastest moving water gradually shifts towards the outer bank as it moves around the bend, and there is a compensating creep of the slower moving water near the bed of the channel towards the inner bank."

The formula for computing the difference of elevation of the water surface between the inside and outside of a bend is given. It is

$$\text{Difference in elevation of Water Surface} = \frac{V^2 b}{gR}$$

in which v = velocity of water
b = width of stream
R = radius of curvature at center of stream.

W. F. Durand. Hydraulics of Pipe Lines. D. Van Nostrand Co., New York, 1921, 271 p. illus.

"The loss is due to the work required to bring about the readjustment of the lines of stream flow." He then gives Weisbach's coefficients, also data of Williams, Hubbell and Fenkell, Brightmore, Schoder and Davis.

"The trend of the investigation indicates, as would be expected, a general decrease in the loss with increasing values of the radius of the bend."

Harrison P. Eddy. Effect of Curvature Upon Flow in Open Channels. Eng. News Record, v. 87, 1921. pp. 516-517.

Eddy does not believe in the application to open channels of data obtained on closed pipes under pressure. He mentions Humphrey's and Abbott's work and Markmann's, Fuller's and Morrell's papers. He also gives a table of Kutter's n for curves and tangents in various channels.

"It appears probable that in artificial channels for which the coefficient n in Kutter's formula would not exceed about 0.025 for a straight channel, an addition of from 0.003 to 0.005 should be made to the coefficient for sections in which the amount of curvature is considerable."

E. Bayerhaus. Die Wirkungen einer Krummung in Offen Wasserlaufen auf Bewegungsorgang und Bettgesyaltung. Zeit fur Bauwesen, v. 72, 1922, pp. 156-163.

Bayerhaus discusses the principles on which the actual movement of water in bends takes place, then gives the results of some experiments in a smooth cement channel without sand. He gives the shape of the water surface in the bend and the distribution of velocities.

"The phenomenon observed in nature that on outer side of the bend the velocity is greater than on the inner side is therefore merely the consequence of the transportation of the water filaments that finally takes place and would not be possible if (as the result of friction on the bottom) at the very entrance into the bend the velocity of the different layers lying over one another did not vary. Since however the transportation takes time and the water keeps flowing further, then the increase in velocity on the outer as against that on the inner side of the bend only occurs after the water has run for a distance through the bend, thus rather near the end of the bend and beyond it, whilst at the beginning of the bend the earlier observed phenomenon of an increase of velocity at the inner side of the bend and decrease of velocity at the outer side must be apparent. The two phenomena are strikingly confirmed by experiments in the Experiment Institute for Hydraulics and Shipbuilding in Berlin."

"In consequence of that the middle of the stream (with the current) at the beginning of the bend runs at first almost in a straight line and the continues in a steadily increasing curve. From that it follows: (1) that finally the curve must become sharper than corresponds to the course of the shore and (2) that the curving of the path of the middle water filaments continues far beyond the end of the curve in the shore, since the change of direction which was held back at the beginning must be finally made up. That explains the well known fact that the erosion of a bank first sets in toward the end of a bend and generally extends far beyond the end of the curve in the shore."

"That causes here in accordance with what was explained above a strong impulsion of the more slowly flowing bottom layer toward the inner side of the bend (therefore here toward the right bank). The sand torn out of the bottom by the increased currents moves spirally toward the protruding bank and settles where

E. Bayerhaus. (Cont'd)

the current by reason of the excessive centrifugal force of the sharp bend can no longer follow but detaches itself from the shore."

"The cross current caused thereby takes the sand in connection with the longitudinal current in a spiral movement away from the left (outer) bank and forms a deep channel out of which the sand at the end of the cross current is carried over to the opposite shore and there part of it is caught in the center current running toward the bank and deposited in the above mentioned shore at and below the bend in the right bank."

Velocity measurements were made by means of a Pitot tube in an open channel with about a 125-degree bend with the approach tangent to the bend about 150 cm. long and the discharge tangent 400 cm. long. The channel had a top width of 60 cm. (23.6 inches) and depth, 14 cm. (5.5 inches); side slopes were $1\frac{1}{2}$ to 1. Depth of flow in tangential part of channel about 10 cm. (4 inches).

Average velocity in tangential section about 140 cm. per second and in bend (on inside of bend) about 150 cm. per second. Velocities taken 3 millimeters under the surface.

"If for instance the radius of curvature on the inner side of the bend is half as great as that on the outer side, then the velocity inside must be twice as great as outside. The observations we have of flow through a smooth channel agree essentially with this (not counting the effect of friction on the walls and the slight depth at the edges.)"

"The earlier mentioned rotatory turning movement in flowing through a bend in consequence of which at the outer side of the bend the upper layers of water rush downward and on the inner side those at the bottom ascend is shown by the fact that on the outer side of the bend the velocities at the bottom are no longer smaller but in spots even greater than those at the surface."

H.W. King and C.O. Wisler. Hydraulics. John Wiley & Sons, New York, 1922, Ed. 3, 292 p. illus.

The authors give the deductions drawn by Fuller and give a table for coefficients in the formula

$$h_B = K_B \frac{V^2}{2g}$$

for various velocities and different radii.

F.W. Medaugh. Loss of Head due to Bends. Engin. and Constr., v. 58, 1922, p. 354.

Medaugh states that previous experimenters neglected to consider eddy losses occurring above the bend. He states that his experiments on a "number of carefully prepared bends of rectangular section indicate that eddying does take place above the bend."

Curtis McD.Townsend. The Hydraulic Principles Governing River and Harbor construction. The McMillan Co., New York, 1922, 189 p. illus.

Townsend mentions Gockinga's article and gives Gockinga's equation for determining the difference of elevation in the water surface.

"These theoretical computations have been confirmed by observations on the Mississippi River, where differences of head of about 1 foot have been observed on opposite banks of bends, with mean mid-section longitudinal slopes of about 0.4 foot per mile."

He then mentions the heliocoidal flow of water in bends as demonstrated by Thomson and states "The direction and velocity of such currents on the Dnieper River were measured by M. Leliavski as described in a paper presented to the Sixth International Navigation Congress held at the Hague in 1894. From the results of his experiments, Leliavski came to the conclusion that in a bend surface currents converge toward the concave bank, along which a stream of water flows to the bottom of the river, thence move to the convex side in divergent currents, and then gradually rise to the surface. He found not only that these currents have sufficient force to cause caving of banks of clay, sand or gravel, but he found evidence also that they had caused erosion in the lava rock which forms the bed of the river at the Dnieper rapids. The greater depth was found where the greatest number of surface filaments of water converge on the concave bank. If the radius of curvature is uniform, this point occurs at some distance below the middle of the bend."

"The influence of a circular band, moreover, affects the straight sections of a conduit for a considerable distance above and below it. Below the bend, it is evident that a considerable distance is required to transform the helicoidal motion again into a rectilinear motion, since this transformation depends on the inertia and the frictional resistance of the particles of water."

Kazuo Kumabe. An Experiment on the Flow of Water Through a Circular Bend of Rectangular Section. Soc. Mech. Engineers, Tokyo, Japan, v. 26, no. 78, June, 1923, pp. 49-67.

Experiments were made in July, 1921, at the Tokyo Imperial University. The rectangular pipe was made of wood covered with glass, channel 2.96 inches wide and 1.56 inches deep. The mean radius of curvature of the bend was 4.5 inches. Velocity measured by getting discharge over a 60-degree triangular-notched weir. Pressure distribution around bend was measured. Bend was 90-degree with approach tangent 27 inches long and discharge tangent 115 inches long. Seven piezometers 1/8 inch in diameter placed 1/2-inch apart in floor at each of seven sections on the bend and seven sections on the two tangents. Piezometers also placed in inside and outside walls at each section.

Seven tests were made with velocity range from 1,97 to 4.71 feet per second.

Kumabe then derives a formula to compute the differences of pressure in the section midway around the bend and his computed values check the observed values very closely.

"The real loss due to the bend is perhaps much smaller than those reported to be, because in the long run, the energy once apparently consumed will be recovered."

"The loss of head due to a bend could be minimized if the center line is made parabolic."

E.H. Lewitt. Hydraulics. Sir Isaac Pitman & Sons, Ltd., London and New York, 1923, 261 p. illus.

Mentions James Thomson's article and quotes his reasoning.

R.A. Monroe. Tests of Friction Losses on Large Penstock. Engin. News-Record, v. 91, 1923, pp. 598-599.

Mentions the fact that no satisfactory data for loss of head in bends in large pipe are available.

Rene Koechlin. Mecanisme De L'eau. Paris, 1924, p. 123.

Considers winding flow of rivers.

H.G. Acres. The Generation of Hydro-Electric Power in Canada. Jour. Canada Engin. Inst. July, 1924, p. 391.

"The theory of this design (of bends) is more conjectural than mathematically precise and is briefly to this effect; (a) that the loss induced by the deflection of the stream lines will be a minimum if the agency producing the disturbance, namely, the curve itself, is made as short as possible, and (b) that the turbulence induced by the change in direction of flow will be reduced and more or less localized if extra lateral space is provided at the seat of disturbance, so that whirls and eddies may be quickly dissipated, instead of being forced forward into the straight section of the waterway. The first requirement is met simply by making the radius as short as possible, and the second by making the curve equiradial."

F.W. Medaugh. Elementary Hydraulics. D. Van Nostrand Co., New York, 1924, 144 p., illus.

Shows Schoder's tests that high velocity is on the outside of the pipe at the end of the bend. Gives photograph of gradient of tests made by himself. Mentions experiments of Williams, Hubbell and Fenkell, Brightmore and Davis.

Lynn Perry. Tests of Loss of Head in Standard Elbows and Tees. Engin. News-Record, v. 92, 1924, p. 940.

Gives diagrams for determining loss of head in elbows and tees. Uses different values of K in the formula

$$H = K \frac{V^2}{2g}$$

for computing the losses in different elbows and tees.

J. Zorn. Over de berekening der drukverlieze in bochtstukken. Waterstaats-Ingenieur, v. 12, 1924, pp. 343-347, illus.

Old formulas of von Weisbach have proved unreliable and a method based on later investigations by Fuller are given for calculation of pressure losses in a fluid passing through an elbow. Table giving actually measured losses at power plant of Tyssadal (Norway), proves greater accuracy of Fuller formulas. Gives charts for calculating losses according to both methods. (Abstract from Engineering Index.)

John Eustice. Flow of Fluids in Curved Passages. Engineering. v. 120, 1925, pp. 604-605. (Article reprinted in Water Works, April, 1926, pp. 178-180.)

"The total loss of head which occurs when a fluid is flowing in the curved part of a pipe, is made up of frictional losses caused by the flow close to the surface of the pipe together with the losses caused by centrifugal force which, in a circular pipe, generates vortices in the fluid and these vortices augment the effect of viscosity; or expressed in another way: If a curved pipe is interposed between two straight pipes, the increased resistance to flow appears to be due to a loss of momentum consequent on changes in direction of the motion of the fluid in the curved part of the pipe, vortices being formed which, after the fluid leaves the curved pipe, continue in the straight pipe. The total loss of head apparently bears some relation to the ratio of the radius of curvature, R, to the diameter D."

Prof. Eustice apparently believes in the "rebound theory." If the fluid passes in line with the discharge tangent on its last rebound, the resistance will be a minimum. The ratios of R/D which give the minimum resistance are 2.91, 12.63, 28.9 and 52.0.

A.H. Gibson. Hydraulics and Its Applications. D. Van Nostrand Co., New York. Ed. 3, 1925. 801 p. illus.

For loss at bends in pipes, he mentions Brightmore's data, gives Weisbach's formula. Gibson apparently made some tests on elbows of rectangular section 1/2-inch by 1 inch, which showed the loss to be proportional to V^2 for all velocities up to 22 feet per second. Gibson gives his coefficients in comparison with Weisbach's. He then discusses Alexander's tests and also William's tests and gives coefficients obtained by Schoder. He mentions experiments made at Yorkshire College (Engineering, Sept. 25, 1896, page 390).

In flow around river bends he gives Prof. James Thomson's theory and experiments.

George E. Russell. Hydraulics. Ed 3, Henry Holt and Co., New York, 1925, 447 p. illus.

Mentions Schoder's, Williams, Hubbell & Fenkell, Brightmore and Alexander tests. Quotes W.E. Fuller's formula,

$$\text{Head lost} = KV^{2.25}$$

and gives values of K for various values of radii in feet.

Fred C. Scobey. Flow of Water in Tulsa 60-inch and 54-inch Concrete Pipe Line. Engin. News-Record, v. 94, 1925, 894-897.

Gives loss of head due to bends in these two pipes. Gives loss in two ways; first, loss per degree of bend, second total loss per bend.

Tests made on a 6.6 mile reach of pipe containing 29 bends, totaling 455.7 degrees.

T. Farrance Davey. Velocity at Tangents of Curves of 36-inch Pipe. Engin. News-Record, v. 97, 1926, p. 905.

Shows velocity traverses taken at tangent of a 36-inch pipe bend for three different quantities of flow.

H.J. Hughes and A.T. Safford. Treatise on Hydraulics. The MacMillan Co., New York, Rev. Ed. 1926, 331 p. illus.

This book gives a digest of the Williams, Hubbell and Fenkell tests, the Saph and Schoder tests, Alexander's and Brightmore's experiments and also Davis' experiments at Wisconsin University. Gives the coefficients the various experimenters obtained and also Alexander's formulas. States no satisfactory solution for loss of head in bends in open channels has yet been offered.

Joseph N. LeConte. Hydraulics. McGraw Hill Book Co., New York, 1926, 348 p. illus.

"The loss in a bend occurs not only in the bend itself but in the straight portion beyond the limit of the bend. Furthermore, the loss attributed to the bend may be considered the excess of loss over and above what would occur in an equal length of straight pipe."

He gives no formula but mentions Murray's and Ashley's experiments in the University of California hydraulic laboratory on globe valves in which they determined the coefficient K in the formula

$$H = K \frac{V^2}{2g}$$

H.K. Barrows. Water Power Engineering. McGraw Hill Book Co., 1927, 734 p. illus.

States that in the canal bends on the Chippewa project of the Ontario Hydro-Electric Commission, the loss was reduced to a minimum by making the curve

H.K. Barrows. (Cont'd) as short as possible, and by providing extra lateral space at the point of disturbance so that eddies may be dissipated before going forward into the straight section of the canal.

William P. Creager and Joel D. Justin. Hydro-Electric Handbook. John Wiley & Sons, Inc., New York, 1927, 897 p. illus.

Gives diagrams for values of K in the formula

$$H = K \frac{V^2}{2g}$$

for various ratios of radius of bend to diameter of pipe.

Loring Wirt. New Data for the Design of Elbows in Duct Systems. General Electric Review, v. 30, 1927, pp.286-296.

Describes tests measuring losses by compressed air in elbows with round and square corners. Finds pipe with square corner has less loss than round corner. States best way to reduce loss is to use large values of radius ratio, R/D, radius over height of pipe, and aspect ratio W/D, with width of bend over height of bend. In case it is necessary to use sharp bends, then direction blades are required to reduce the excessive losses. Novel methods used to get direction of filaments of flow of air in the models. Results show two secondary currents similar to those obtained in Iowa tests.

Anonymous. Power, v. 66, 1927, p. 61.

Editorial discussing Loring Wirt's article in the General Electric Review.

Dr. A. Stodola. Steam and Gas Turbines. Translation by Dr. Louis C. Lowenstein, Ed. 6, 1927, v. 1, p. 168.

Discusses secondary or spiral flow in bends. States Lolonger's dissertation (Zurich, 1913) "Untersuchung des Druck und Stromungsverlaufs in Schaufeln" contains a very complete investigation of secondary flow in pipe bends. Secondary flows are produced by the effect of friction against the lateral surfaces of the channel. Mentions that velocities are much higher on the inside of the bend than on the outside, and that this velocity difference produces violent eddies in the flow with corresponding losses of kinetic energy.

W. R. Dean. Note on the Motion of Fluids in a Curved Pipe. London, Edinburgh and Dublin. Phil. Mag. & Jour. of Sci., v. 4, 1927, pp. 208-223, illus.

In this paper steady motion of incompressible fluid through pipe of circular cross-section which is coiled in circle is considered. (Abstract from Engineering Index.)

W. D. Edwards. Pressure Losses in Air Ducts. Heating and Ventilating Mag., Feb. 1928, p. 69.

Dr. Ing. D. Thoma. Mitteilungen des Hydraulischen Institute Der Technischen Hochschule. Munchen, Heft 2. R. Oldenbourg, Munich and Berlin, 1928, p. 72 illus.

Article in above paper by Dr. Ing. G. Vogel on "Experiments for Determining the Loss in Right-Angle Pipe Tees" gives experimental data and coefficients for determining those losses.

Article on "New Experiments on Pressure Loss in Pipe Bends" by Dipl.-Ing. A. Hoffmann show lower losses than those found in experiments of Williams, Hubbell, and Fenkell; Black, Brightmore, and Schoder.

Article on "Losses in Different Forms of Elbows" by H. Kirschbach, Studien-Professor, shows data on different forms and shapes of elbows.

W. R. Dean. The Stream-line Motion of Fluid in a Curved Pipe. London, Edin., and Dublin Phil. Mag. and Jour. of Sci., v. 5, 1928, pp. 673 to 695, illus.

Analysis to find relation between pressure gradient and rate of flow through curved pipe.

__________ Fluid Motion in a Curved Channel. Proc. Royal Soc., v. 121 No. A-787, 1928, pp. 402-420.

Theoretical investigations of stability of flow in curved pipe is certain to be a matter of great difficulty, therefore, simplified form of problem stability of flow under pressure through curved channel is considered; author shows that flow in curved channel may become unstable for small disturbances; offered as tentative explanation of known absence of marked velocity of flow in curved pipe.

E.D. Edwards. Change in Duct Design Saves 400,000 Kilowatt Hours a year. Power, v. 68, 1928. pp. 677-678.

E. D. Edwards. (Cont'd)

Describes the revision of the flue duct design in the East Peoria Station of the Illinois Electric Power Co., according to suggestions made by Loring Wirt. States savings in auxiliary power through the change in design was equivalent to a saving in coal of 347 tons a year.

B. L. Spivak.

Square Bends Reduce Elbow Losses in Ducts. Power, v. 69, 1929, pp. 55-56.

Discusses article by Edwards and describes flow of air around bends. Shows how dimensions of bend affect the efficiency and gives best ratios to follow and efficiency gained. No formulas are given.

Anonymous.

Verlustminderung in rechtwinkligen Kruemmern. Die Waerme, Berlin, v. 52, 1929, pp. 792-793. (See Mech. Engin., January 1930, p. 101.)

Based on numerous and detailed tests on rectangular pipe bends. Data given on velocity distribution, pressure curves and losses. States that losses due to change in direction of flow can be reduced to a minimum by shape which deviates from that of standard bends.

H. Richter.

Der Druckabfall in gekruemmten glatten Rohrleitungen. Forschungsarbeiten auf dem Gebeite des Ingenierwesen, Berlin, No. 338, 1930, 30 p. illus. (See Mech. Engineering, March, 1931. p. 250.)

Report of study at Freiberg on glass and drawn copper pipe of 20 to 40 mm. in diameter, both straight and curved. Pressures were measured to within 1/1000 mm. of water column. Loss of head was determined as a function of Reynold's number, angle of deflection, radius of curvature, and diameter of pipe. Formula derived was found in agreement with observations.

Hydraulic Laboratory Practice. Amer. Soc. Mech. Engin., 1929, 868 p. illus.

Prof. O. Franzius, Dr. Eng'g Streck, & Dr. Eng'g Hinderks, The Experimental Institute for Hydraulics and Foundations at Technical University of Hanover, p. 612.

Pictures showing secondary flow in bends and elbows.

A. Hofmann.

New Investigations to Determine the Pressure Loss in Pipe Bends. Hydraulic Institute of the Technical University at Munich, p. 472.

A. Hofmann. (Cont'd)

Tests conducted on split bronze bends, milled extremely accurate. "....inside diameter of 43 mm. (1.7 in.) the radii of the center lines were 1, 2, 4, and 6 times pipe diameter."

After measurements on the bends were completed, the bends were replaced by equal lengths of straight pipe equal to extended length of corresponding bend and measurements taken again.

Gives chart of K vs. R/D, where K is in the formula

$$H = \frac{KV^2}{2g}$$

for following experimenters, Weisbach; Davis, Williams, Hubbell, and Fenkell, Brightmore, Schoder, Hofmann and Alexander.

H. Kirchbach.

Losses in Various Forms of Elbows. Hydraulic Institute of Technical University of Munich, p. 473.

Full page sketches show K for various elbows. Describes tests on pipe elbows 43 mm. (1-7-in.) in diameter.

A. H. Gibson.

Encyclopaedia Britannica, 14th ed., 1929, v. 11, p. 971.

"Best radius in practice is from 2.5 to 5 times the pipe diameter. For such bends the loss is given sufficiently nearly by

$$\frac{0.3V^2}{2g} \text{ feet.}$$

"Where the bend is carried around an angle θ less than 90-degrees, the loss is very nearly proportional to θ^2

H. Lorenz.

Der Widerstand von Rohrkruemmern. Physikalische Zeit., v. 30, 1929, pp. 228-230, illus.

Theoretical mathematical discussion of conditions of flow and optimum relation between diameter of pipe and radius of curvature.

G. I. Taylor.

The Criterion of Turbulence in Curved Pipes. Proc. of Royal Soc., v. 124, No. A-794, June 4, 1929, pp. 243 to 249, illus.

Account of the experiments at Cavendish Laboratory; conclusion reached by C.M.White, that higher speed of flow is necessary to maintain turbulence in

G. I. Taylor. (Cont'd) curved pipe than in straight pipe, is verified directly; in pipe bent into helix, diameter of which was 18 times that of cross-section, steady streamline motion persisted up to Reynold's number 5830, i.e., 2.8 times Reynold's criterion for straight pipe.

H. Nippert. Ueber den Stroemungsverlust in Gerkruemmten Kaenalen. Forschungsarbeiten auf dem Geibeits des Ingenierwessens, heft 320 , Berlin, V.D.I. Verlag, 1929, 67 p. illus. See also abstract in V.D.I. Zeit., v. 73, 1929, pp. 1391 to 1392, 1932.

With the support of various German engineering and physical societies and firms, author has made extensive investigation of flow of water in curved pipes and channels; he reviews previous investigations of stream flow and losses in bends, describes various factors that determine them, and investigates very fully effect of certain of these factors by careful experiments; results are given in graphic charts and in photographs of flow in open channels. Eng. Soc. Library.

M. Kawada. On the Measurement of Pressure and Velocity of Water Stream In a Curved Pipe. Soc. Mech. Engin. Jour. (Tokyo) v. 32, 1929, pp. 454-466, illus. (In Japanese.)

Result of measuring pressures and velocities of flow of water in pipe bend which is curved at right angle in horizontal plane; as preliminary experiment, motion of colored water streamlines in glass bend pipe had been observed. (Abstract from Engineering Index.)

Dr. Ing. H. Nipper. Neuere Versuche Über den Stronungsvorgang in Gekrümmten Kanalen. Der Bauingenieur, Heft 5, January 31, 1930, pp. 76-79, illus.

Experiments conducted at the Institute of Hydrodynamics, Technische Hochschule, Danzig in 1925-26 on 90-degree and 180-degree bends. Models of cast-iron, generally with a rectangular cross-section. Inner and outer walls changeable so the influence of the inner and outer curvatures could be investigated separately. Furthermore the entrance section to outlet section could be changed. Apparatus so set up that it was possible to compare the times definite quantities of water passed through different bends. From these time values the resistance values of the tested bends could easily be calculated. Pressure distribution on the bend walls was measured and velocity distribution in the entrance and outlet sections was measured. Pressure curves are similar to those obtained on Iowa tests.

Dr. Ing. H. Nippert. (Cont'd)

Losses are not caused by one single factor. They are caused by (1) the wall friction, (2) the formation of whirls and the consequent transversal flow. When the ratio of the radius of the bend (measured to the center line) over diameter of pipe exceeds 3.5, the losses are caused principally by walls friction. The transversal flow is then negligible.

The losses in bends are dependent upon the following factors: (1) The ratio of the curvature to the breadth or diameter of the channel, r/b, r/d. (2) The cross-section sequence in the direction of flow. (3) The shape of the cross-section or ratio of height to breadth, h/d.

The loss coefficient is large for bends of small radii as compared to the breadth (b) or diameter of pipe. For small ratios of r/b, the losses are great but as the ratio of r/b increases, the coefficient decreases rapidly. For greater values up to r/b equals 4.0, a decrease in the coefficient can be observed. A further decrease of this ratio however does not produce an appreciable gain.

Bends with convergent cross-section in the direction of flow show smallest loss coefficients. The losses increase somewhat for bends with equal entrance and outlet sections, and increase considerably for bends which are divergent in the flow direction.

A minimum value of loss coefficient is obtained when depth of channel over width h/b equals 2.0. The loss coefficient was found to vary with the square of the velocity. The pressure needed to pass a certain flow through a pipe line can be considerably decreased by using a slightly widened section in the vortex of the bend. This may be of some significance in all practical cases, where the available pressure head is to be used to its limit, for example in the draft tubes of water turbines, in the filling and draining channels of lock chambers, etc. No analytical relation between the measured loss coefficients and the pressure differences on the inner and outer walls could be found.

R. Bambach.

Ploetzliche Umlenkung (Stoss) von Wasser in geschlossen unter Druck durchstroemten Kanaelen. Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, No. 327, 1930, 28 p. illus.

Investigation of loss of head in pipe bends conducted at Hanover Institute of Technology. Results show that the generally accepted theoretical formulas

R. Eambach. (Cont'd)

are not correct. Many pressure observations and velocity contours were taken. The spiral motion was investigated by means of strings.

G. J. Klein, K. P. Tupper, and J. J. Green. The Design of Corners in Fluid Channels. Canadian Jour. of Research, v. 3, September 1930.

Describes experiments on flow of air around bends. Shows details of blade turns used. States that vanes for corners are very effective in reducing losses.

H. Nippert.

Abstract of article "Reducing Friction Losses in Rectangular Ducts". Power, v. 7, 1930, pp. 918-919.

In Power for January 8, 1929, B.L. Spivak had an article on reducing losses in bends. This article was abstracted in the March 2 issue of "Die Warme" and was discussed in the October 19 issue. The following review is an abstract of the latter article:

"The pressure changes and the spiral motion in bends is discussed. Losses in bends will be reduced to a minimum when the following conditions are fulfilled:

(1) When in long radius bends, the maximum ratio between the radius and the width of the duct is 4.5.
(2) When the cross-sectional area is increased from the beginning of the bend to the midway section and then gradually diminished towards the end.
(3) When the maximum ratio between the duct height and the width is 2.0. (For rectangular ducts).
(4) The high losses in square bends may be reduced by the use of guide vanes.

H. Chatley.

Curvature Effect in Alluvial Channels. Engineering, February 13, 1931, pp. 196-197 and February 20, 1931, pp. 260-261.

Mentions the work of Boussinesq, Fargue, and Ripley and states that their theories do not agree with the sections of the Whangpoo River. Goes into the mechanics of flow around open channel curves and gives a formula showing relation between depth, radius of curvature, and angle of curvature. Describes secondary in bends and transverse in bends and gives Gockinga's formula for computing the transverse slope.

S. Erk.

Forschungsarbeiten auf dem Gebeite der Technischen Hydrodynamik. Forschung auf dem Gebeite des

S. Erk. (Cont'd) Ingenieurwesens, v. 2, 1931, p. 19-27, illus.

Reviews progress made on loss of head in straight pipes with constant cross-section, in curved pipes and divergent and convergent conduits, and discusses other phases of hydraulic researches. Gives a brief résumé of Nippert's work, of Richter's tests, and Bambach's experiments on closed channels with constant cross-section and with sudden turn of flow.

Robert William Angus. Hydraulics for Engineers. Sir Isaac Pitman & Sons, Ltd., Toronto, 1931, 300 p. illus.

Mentions tests by Professor Giesecke at University of Texas and reported in bulletin No. 2712, March 1927, under title, "The Friction of Water in Elbows."

Tests made on screwed fittings 45-degrees and 90-degrees, long and short radius, 1 inch to 3 inches diameter.

Also speaks of Fuller's paper.

Sabin Crocker and Arthur McCutchan. Frictional Resistance and Flexibility of Seamless-Tube Fittings Used in Pipe Welding. Fuels and Steam Power Paper No. 53-17, Trans. Amer. Soc. Mech. Engin., v. 53, 1931, pp. 215-245.

Discusses as one phase of their subject the pressure drop or loss of head in bends or elbows. Mentions Busey's paper in which is given a diagram showing that in air ducts the loss is reduced to a minimum when R/d equals 3/2 and that nothing additional is gained by making R/d equal to 2.0.

Edwards' article on the flow of air through bends shows that no appreciable reduction in pressure drop is shown for a mean radius of bend greater than 1.5 times the diameter of the pipe.

The tests of Schoder, Davis, Brightmore and Pound on bends using water for the fluid were reviewed and the results of their tests compiled in a diagram. The loss due to friction in the bend was deducted from the total loss and the resultant amount is the loss formed by the bend itself. The graphs showed a sharp drop in loss as R/d increases from less than unity up to a value of 1.5 or 2.0. When R/d exceeds a value of 2 or more the decrease in the loss due to the bend is very slight and when the values of R/d exceed 5, the loss due to the bend appears to increase. State that Williams, Hubbell, and Fenkell found in their tests on

Sabin Crocker and Arthur McCutchan.
(Cont'd) large size pipes that the loss increases when R/d exceeds 2.5 and the minimum loss occurred when R/d was equal to 2.5.

Discussion of the paper by various engineers. Giesecke's and Eustice's work quoted.

Eugene E. Halmos. Loss of Head at Branches Determined for Water Pipes. Engin. News-Record. v. 108, 1932, p. 684.

Experiments conducted at hydraulic laboratory of Technical University of Munich, Germany. Description of experiments and analyses of data obtained are contained in paper by Emil Kinne, published as vol. 4, (1931) of Proceedings of Hydraulic Institute of Technical University, edited by Dr. Thoma.

J.W. MacMeeken. Turbulence in Centrifugal Pumps. Hydraulic Paper No. 54-4. Trans. Amer. Soc. Mech. Engin., v. 54, 1932, pp. 47-63.

Describes method of computing head losses in river bends when the velocities and radii are known. Gives the formula

$$Z_{1,2} = \frac{(1 + P_1)\ V_1^2 - (1 + P_2)\ V_2^2}{2g} + \int_{L = 0}^{L = 1-2} \frac{V^2(1 + P)dL}{C^2 r}$$

in which

$Z_{1,2}$ = drop in water surface in an open channel between two points.

V_1 = mean velocity of channel near beginning of bend at point 1.

V_2 = mean velocity of channel near end of bend at point 2.

V = mean velocity of channel.

C = Chezy coefficient from the formula

$$C = \frac{157.8}{1 + \frac{Y}{\sqrt{r}}} \quad \text{(Bazin's 1897 formula)}$$

r = mean hydraulic radius.

J. W. MacMeeken. (Cont'd)

(1 + P) coefficient of local distribution calculated from the curvature of the stream line boundaries of the channel and the position of the first and second centroids with reference to the local center of curvature.

"The value of (1 + P) was obtained in two ways: First by gaging the stream at sections 1 and 2 a short distance apart, recording all the values of the velocity at a uniformly distributed number of points and finding both the average velocity and $\sum V^2$ number of observations, then

"The value of (1 + P) was next computed by the formula

$$(1 + P) = \text{ratio } \frac{\sum V^2 / N}{(\text{Average } V)^2}$$

and the same values resulted.

"The value of the coefficient (1 + P) varied from 1.20 to 1.70 in curved channels to 1.06 in straight channels adjoining bends in the stream."